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its place had been accurately pointed out to them, but not with satisfactory certainty. I myself always made the observations best if I came to the telescope with fresh eye. After some minutes the eye became usually so exhausted that I was obliged to give it some rest to enable me to see the object. Some of the later observations on separate evenings are therefore of inferior accuracy.

The difficulty of the observation of the companion of *Procyon* will of course be unequally greater than that of the companion of *Sirius*. In our latitudes this difficulty is somewhat equalised by the low position of *Sirius*: for the companion of which I have looked in vain on many evenings in the years immediately after its discovery by Alvan Clark, until at last favourable atmospheric conditions enabled me to see it distinctly. But after I had once recognised it, and thereby followed the modifications of its appearance with tremulous images, it is now only when there is a high degree of tremor that I cannot see it, at least by glimpses, with certainty. These experiences have doubtless been of use to me in the recognition of the companion of *Procyon*, and give me also full confidence in the accuracy of my perceptions, independently of the corroborating observations of Lindemann and Ceraski. Notwithstanding this, however, I willingly acknowledge that a confirmation of them by other observers at other places with other instruments, would be very welcome to me.

On a Method of finding the Parallax of Double Stars, and on the Displacement of the Lines in the Spectrum of a Planet.

By Professor C. Niven.

(*Communicated by W. H. M. Christie, Esq.*)

§ 1. The idea of Doppler that the motion of the fixed stars towards or from the Earth must exercise an influence on the nature of the light which we receive from them, though it has not served his purpose of explaining their colours, has yet not been without fruit. For although, in general, no great change of colour can be looked for in this direction, we may nevertheless expect to find traces of the effect in a displacement of the fixed lines of the spectrum. The displacement of the line F of hydrogen for a velocity of 20 miles per second amounts to nearly $\frac{1}{12}$th of the distance between the constituents of the double line of sodium, and is towards the violet or red end, according as the star is approaching or receding from us. Now displacements of this magnitude are fully within the reach of modern instruments, and accordingly Mr. Huggins has succeeded in estimating the velocities of separation or approach of a number of the brighter stars.

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At the meeting of the British Association in Edinburgh, Mr. Fox Talbot suggested the application of the spectroscopic method to the determination of the distances of certain double stars. It is manifest that if we know the form of the orbit of a binary system, and its position with regard to the plane of the heavens, or in other words, of its apparent orbit, we require only one linear element to determine its absolute magnitude. And if this be given, the parallax of the system can be readily found; and, moreover, if we suppose, as there is every reason to do, that the law of gravitation holds good for such star systems, we have the means of determining the sum of their masses.

The linear element required would be supplied by an observation of the component in any direction of the relative velocity of the two members of the system, as, for instance, the component along the line joining the Earth with the star. For this depends on the orbital motion only, and not on any proper motion which the system as a whole may possess.

The difficulties in the way of effecting this determination arise from three sources:—the faintness of the stars, the smallness of their angular separation, and possibly also from the component of velocity under consideration being too small to be capable of measurement. With regard to the first two of these, it may be observed that Mr. Huggins has recorded in the *Philosophical Transactions* for 1864, that he was able to examine separately the two components of *β Cygni*, and of *α Herculis*. Now the magnitudes of the two components of the former of these are respectively 3 and 7 (Smyth), while those of the latter are $3\frac{1}{2}$ and $5\frac{1}{2}$, and are separated by an angle of $4''\cdot6$. It is therefore inferior in both respects to *Castor*, the two members of which are of magnitudes 3 and $3\frac{1}{2}$, and which were separated at the beginning of the year 1869 by a distance of nearly $6''$.

With regard to the third difficulty, it is only in one case, that of *α Centauri*, that we can speak with any certainty. Adopting the parallax of $0''\cdot913$, and the old elements calculated by Captain Jacob, I find the greatest velocity of approach to amount to 12·556 miles, and the greatest velocity of separation to be about 25·85 miles per second. With the elements more recently given by Mr. Powell, the greatest velocities of approach and separation are respectively only 6·82 and 20·41 miles; while about the present epoch the velocity is one of separation amounting to about 17·43 miles per second.

Under these circumstances there seems good ground for believing that the observations required are not beyond the reach of the present instrumental means; I have therefore thought that some service might be done by removing the geometrical obscurities of the problem, and showing what the formulæ are which serve, by the aid of the observations, to determine the parallax.

The general result of the investigation is a relation of the

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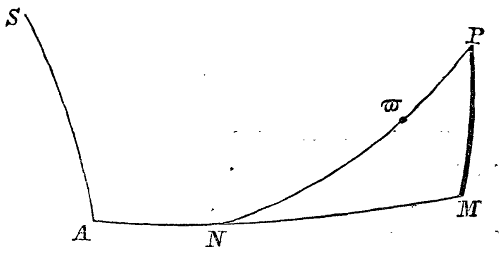
form $p\,\mathrm{W}=\mathrm{A}\cos\theta+\mathrm{B}$; wherein θ is the position of the secondary measured on the relative orbit from the node, p the parallax of the system (in seconds), W the component of the relative velocity measured *from* the Earth along the line to the system, and A and B two constants to be calculated in each case from the elements of the orbit. Here, as throughout the present paper, W is taken to be positive when considered as a velocity of separation, and negative when one of approach.

Some general deductions, which can be made from the above formulæ, will be considered in their proper place.

In Articles 5 and 6 I have considered the effect which the motion of a planet has on the lines in the spectrum of the light reflected from it.

§ 2. In considering the motion of a binary star system we take for plane of reference the tangent plane to the heavens, for the initial line that in which this plane is cut by a declination circle, and the direction of measurement to be north—following—south—preceding. It seems sufficiently established that the relative motion of the system may be represented by supposing the secondary component to trace out an ellipse in a plane inclined to the plane of reference conformably with the ordinary laws of elliptic motion, and having the primary as focus.



Let S be the primary, A the origin of measurement, N the node of the orbit, ϖ periastre, and P any position of the secondary; let, also, the arc P M be drawn at right angles to the plane of reference. Denote as usual the elements of the actual ellipse by a, e, N, i, ϖ, and by T the time of revolution in years. Let $\mathrm{N\,P}=\theta$, $\mathrm{N\,M}=\theta$, and $\mathrm{P\,M}=\lambda$.

We thus have, first of all,

$$\left.\begin{aligned}\text{angle of position} &= \mathrm{N}+\theta_1\\ \tan\theta &= \tan\theta_1\sec i\\ \sin\lambda &= \sin i\sin\theta\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(1)$$

To connect the actual and angular mean distances a and α'', we have

$$a:\nabla::\alpha:p\quad\ldots\ldots\ldots\ldots\quad(2)$$

∇ being the diameter of the Earth's orbit.

If we call $\mathrm{S\,P}=r$, we find from the theory of elliptic motion:

$$\frac{1}{r}=\frac{\mu}{h^2}(1+e\cos\overline{\theta-\varpi})$$

$$\frac{dr}{dt}=\frac{\mu e}{h}\sin\overline{\theta-\varpi})$$

$$r\frac{d\theta}{dt}=\frac{\mu}{h}(1+e\cos\overline{\theta-\varpi}).$$

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The motion of the star is compounded of the velocities $\frac{dr}{dt}$ along S P and $r\frac{d\theta}{dt}$ at right angles to it. The component of the former at right angles to the plane of the apparent orbit is

$$\frac{dr}{dt}\sin\lambda, = \frac{\mu e}{h}\sin i\sin\theta\sin\overline{\theta-\varpi},$$

and that of the latter is

$$r\frac{d\theta}{dt}\sin i\cos\theta.$$

Thus, the whole component of velocity, W

$$= +\frac{\mu}{h}\sin i\,(\cos\theta + e\cos\varpi).$$

Now we have also

$$\frac{\mu}{h} = \frac{2\pi a}{T\sqrt{1-e^2}},$$

whence, if W be measured in miles per second, and n be the number of seconds in a year,

$$W.T.n = \frac{2\pi a\sin i.\nabla}{p\sqrt{1-e^2}}(\cos\theta + e\cos\varpi)\;.\;.\;.\;.\;(3)$$

[It is assumed in this result that the motion of the secondary in its apparent orbit is *direct.*]

§ 3. It follows from the expression just found that W attains its greatest values at the two nodes, being a positive maximum at the one and a negative maximum at the other. Of the two that is the greater which corresponds to the node nearest periastre, and this will be positive or negative according as the star in passing through periastre recedes from or approaches the nearest node. At two points situated symmetrically with respect to the line of nodes W vanishes. These points separate the region of the orbit from which W is positive from that in which it is negative, and are determined by $\theta = \pm\beta$, where $\cos\beta = -e\cos\omega$.

If we write p W = A cos θ + B, the two maxima are A + B and $A \sim B$; B represents the mean value for equal angular spaces round the primary.

The annexed Table contains the values of the more important quantities for most of the orbits whose elements are known. Columns 5 and 6 contain the maxima values of p W, and include both magnitudes and signs. In column 8 are found the angles of position corresponding to the points for which W vanishes. The symbol + between the two angles indicates that, for positions of the secondary between the minus, W is positive. The reverse is denoted by the symbol −.

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TABLE I.

Star.	Elements computed by	A.	B.	Greater max. value of p W.	Less max. value of p W.	Log A.	Angles of Position for which p W = o.	
							° ′	° ′
η Coronæ Bor. . .	Mädler . .	·92646	·04704	− ·97350	+ ·87942	$\bar{1}$·966 8285	105 22	− 303 14
ζ Cancri . . .	Mädler . .	·7821	·1281	+ ·9102	− ·65395	$\bar{1}$·893 2505	71 11½	+ 291 44½
ξ Ursæ Majoris . .	Herschel .	1·9528	·4097	+ 2·3625	− 1·5431	·290 6615	153 44	+ 41 50
ω Leonis . . .	Villarceau .	·3608	·2311	− ·5918	+ ·1297	$\bar{1}$·557 2011	174 41½	− 95 40½
p Ophiuchi . .	Herschel .	1·6851	·6502	+ 2·3353	− 1·0349	·226 6103	184 52	+ 69 58
ξ Boötis . . .	Herschel .	4·8088	2·8552	− 7·6640	+ 1·9536	·682 0362	166 58	+ 193 7
δ Cygni . . .	Hind . .	·3381	·09184	+ ·4300	− ·2463	$\bar{1}$·529 0691	91 39	+ 337 9
γ Virginis . . .	Herschel .	·7002	·4566	− 1·1568	+ ·2436	$\bar{1}$·845 2215	183 52½	+ 317 43½
Castor . . .	Herschel .	1·6903	·1669	+ 1·8572	− 1·5234	·227 9533	96 52	+ 309 18
σ Coronæ Bor. . .	Hind . .	·1624	·04147	+ ·2039	− ·1210	$\bar{1}$·210 6979	127 22½	− 274 43½
μ_2 Boötis . . .	Hind . .	·2445	·04719	+ ·2916	− ·1973	$\bar{1}$·388 1834	191 16½	+ 43 25½
α Centauri . .	Jacob . .	17·533	6·0684	+ 23·6014	− 11·4646	1·243 8514	204 57½	− 327 16½
α Centauri . .	Powell . .	12·433	6·2070	+ 18·640	− 6·226	1·094 5392	9 27½	+ 39 8½
ζ Herculis . .	Villarceau .	·9765	·1127	+ 1·08914	− ·8638	$\bar{1}$·989 6616	115 13	+ 313 29

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The great preeminence of *a Centauri* in this list is due, mainly to the largeness of its mean distance caused by its proximity to us. With the aid of the commonly received value of its parallax, I have calculated the two maxima values of W and its value at the present epoch. They are given at the commencement of this paper.

§ 4. Through the kindness of Mr. Gill, who has supplied me with some of Dembowski's recent measurements of binary systems, I am enabled to add the present values of p W in certain cases. The observations do not give directly the values of the distance and position-angle at the present time; but we may approximate to them by assuming their rates of change, as determined by two sets of observations, to remain sensibly constant for a few years. In two cases the observations of Dembowski have been combined with those of Brünnow, made at Dunsink during the years 1868-69.

The distances are given in the table to indicate what stars are more favourably situated for observation :—

TABLE II.

Star.	Position-Angle and Distance.							Value of p W.
	Observed.				Calculated.			
	Epoch.	Position-Angle.	Dis-tance.	Ob-server.	Epoch.	Position-Angle.	Dis-tance.	
		°	″			°	″	
η Coronæ Bor.	1872·44	50·86	...	D	1874·44	61·44	0·975	+ ·3167
	1873·44	56·15						
ζ Cancri .	1872·33	162·84	...	D	1873·33	135·86	1·0	+ ·4432
	1873·33	150·23	0·5					
ξ Ursæ Majoris	1872·33	19·39	...	D	1873·33	338·43	·882	+ 1·0078
	1873·33	358·91						
p Ophiuchi .	1868·90	98·008	4·92	B	...	...	...	+ ·4565
δ Cygni . .	1868·69	349·06	1·52	B	...	...	...	+ ·09307
γ Virginis .	1869·22	164·98	...	B	1873·33	159·01	4·52	− ·4757
	1872·86	160·79	...	D				
Castor .	1869·16	238·97	5·71	B	...	...	...	− ·86162
σ Coronæ Bor.	1868·55	194·048	...	B	1873·33	199·33	3·13	− ·12087
	1872·96	198·08	...	D				
ζ Herculis .	1872·48	173·88	...	D	1873·33	153·62	1·43	+ ·4893
	1873·52	162·49						

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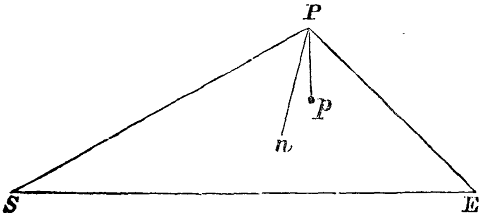
§ 5. *Displacement of the Lines in the Spectrum of a Planet.*—In all the cases, so far as I am aware, in which the effect of the motion of the source of light on the spectrum lines has hitherto been treated, the body has been supposed self-luminous. But it may be readily shown that somewhat similar results obtain when the substance shines either by reflected or transmitted light. To make this clear in the case of reflexion, which is the more important one from an astronomical point of view, let us first consider what would happen if the spectator and source of light were in a line with the reflecting body, and the latter were moving towards us with a velocity v. Supposing any change of phase which may occur at reflexion to be unaffected by the motion of the mirror, it follows that the path of a second luminous crest will be shorter than that of the first by twice the distance which the mirror has travelled during the period of a single undulation. The number of vibrations per second in the reflected light is therefore increased in the ratio $1 + \frac{2v}{V} : 1$, where V is the velocity of light. The wave-length of the light which the spectator receives is shortened in the same ratio.

The effect of the mirror in this case is, thus, double what would take place if the body were self-luminant.

Suppose now, in the second place, that the incidence on the reflector is oblique, and that the latter is moving in any direction in the plane of incidence. Let S be the source, E the spectator, P the reflector, P p the distance moved in time $\delta t = v\,\delta t$, P n the bisector of the angle S P E. Let also $SPE = 2\alpha$, $pPn = \beta$.



Since it is a theorem that, for the point (P) of reflexion at the mirror, S P + E P is a minimum, and therefore stationary, we may still in the new position of the mirror treat p as the point reflexion. By the motion of P the path of the light is shortened by a length $v\,\delta t\,(\cos\overline{\alpha + \beta} + \cos\overline{\alpha - \beta}) = 2\,v\cos\alpha\cos\beta\,\delta t$. The period of vibration and wave-length are, thus, each shortened by a fraction of itself $= \frac{2\,v\cos\alpha\cos\beta}{V}$. The reverse would happen if P moved in the direction p P.

[A very similar formula expresses the effect of any motion possessed by a refracting surface; and it follows that, in transmission through a plate with parallel faces, motion of the latter produces no effect whatever on the lines of the spectrum.]

§ 6. These principles may be applied to the analysis of the light which we receive from the planets. And in showing how this may be done, I shall adopt the restriction that all the planets revolve in the ecliptic, in concentric circles round the Sun.

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The results which we shall thus obtain will not be strictly accurate, but will probably not differ much in their general features from those actually presented in nature. They will, at least, enable us to judge, with more or less truth, of the magnitudes of the numbers with which we have to deal.

Since the magnitudes with which we are concerned are exceedingly minute, we may employ the principle of the superposition of small changes to separate the effect due to the motion of the planet from that of the Earth. Let these bodies be represented as in the former figures by P, E, and let their distances from the Sun be a, b miles, and let their times of revolution be T and t years. P and E are moving at right angles to S P, S E with velocities $\frac{2\pi a}{\mathrm{T} n}$ and $\frac{2\pi b}{t n}$ miles per second, the direction of revolution being from E to P. Denoting these by u, v, it is evident that the joint effect of the motion of both is to increase the wavelength of the light emitted from the Sun by a fraction of itself $= \frac{\mathrm{U}}{\mathrm{V}}$, where

$$\mathrm{U} = 2u \cos \alpha \;\sin \alpha - u \;\sin \mathrm{SEP}$$

$$= \frac{2\pi}{n}\left(\frac{1}{\mathrm{T}} - \frac{1}{t}\right) b \sin \mathrm{SEP} \;.\;.\;.\;.\;.\;. \quad (4)$$

From this formula we gather the general result that, for equal angular distances of a planet for the Sun, the values of U are numerically equal. To investigate it more narrowly, two cases must be distinguished,—

1°. For an *inferior* planet, for which $t \;7\; \mathrm{T}$, U is positive when the planet is east of the Sun, and negative when west of it. At the greatest elongations of a planet U is a maximum and numerically $= u\,(1 \overset{\mathrm{T}}{-} \frac{\mathrm{T}}{t})$.

2°. For a *superior* planet, for which $t \angle \mathrm{T}$, U is negative for positions east of the Sun, and positive for western positions. Its maximum is reached when the distance is a right angle, and it then amounts to $u\,(1 - \frac{t}{\mathrm{T}})$.

The following Table gives the values of U_g for the principal planets calculated with the restrictions above mentioned :—

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TABLE III.

Planet	Ug in miles per second
Mercury	22·24
Venus	8·24
Mars	8·57
Asteroids	13·87
Jupiter	16·76
Saturn	17·68

Queen's College, Cork,
1874, *April* 11.

On a piece of Apparatus for carrying out M. Janssen's Method of Time-Photographic Observations of the Transit of Venus. By Warren De La Rue, D.C.L., F.R.S.

(*Described verbally at the Meeting of the* 10*th of April*, 1874.)

It will be remembered that M. Janssen proposed to take advantage of the preparations which were being made for photographic observations of the transit of *Venus*, by adding thereto a special contrivance by means of which a series of photograms might be taken in rapid succession, when *Venus* had actually entered on the limb of the Sun. It was thus hoped to ensure a more exact time-record of the internal contacts of the planet and the limb of the Sun, than could be obtained by eye-observations.

The general scheme of the apparatus necessary to effect this object was well considered by M. Janssen, and there is little novelty in the form of the instrument I am about to describe; nevertheless, as there are some peculiarities in its construction, and as it contains the fewest possible number of moving parts, I have thought that it might not be unworthy of the notice of the meeting.

The photographic plate to be employed is supported in a cell which rotates on an axis excentric to the axis of the telescope;